(c) Test for convergence the series

(i) $\sum_{n=1}^{\infty} \frac{2^{n-1}}{3^n + 1}$

(ii) $\sum_{n=1}^{\infty} \frac{r^n}{n!}$

6. (a) State D'Alembert's Ratio test for the convergence of positive term series. Use it to test the convergence of the series $\sum_{n=1}^{\infty} \frac{5^n}{n^2 + 5}$.

(b) Test for convergence the series

$$1 + \frac{2}{5}x + \frac{6}{9}x^2 + \frac{14}{17}x^3 \frac{30}{33}x^4 + \cdots$$

(c) Show that the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\log(n+1)}$ is conditionally convergent.

[This question paper contains 4 printed pages.]

Your Roll No.............

Sr. No. of Question Paper : 7041 **K**

Unique Paper Code : 2352203501

Name of the Paper : Elements of Real Analysis

Name of the Course : **B.A. / B.Sc. (P) with Mathematics as Non-Major / Minor**

Semester : V

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory and carry equal marks.
3. Attempt any **two** parts from each question.

1. (a) Let F be any ordered field and x, y, z $\in$ F. Show that if $x < y$ and $y < z$, then $x < z$.

(b) Prove that for all x, y $\in \mathbb{R}$,

$$\min\{x,y\} = -\max\{-x, -y\}.$$

(c) Define an upper bound of a non-empty subset of R. Find supremum and infimum of the following sets :

(i) $\left\{\frac{x+1}{x} : x > 2\right\}$

(ii) $\left\{2 - \frac{1}{n} : n \in \mathbb{N}\right\}$

2. (a) Let F be an Archimedean ordered field, $A \subseteq F$ & $u \subseteq F$. Then $u = \sup A \Leftrightarrow$ for all $\in > 0$ and $x \in A$, $x < u + \in$, and there exists $x \in A$ such that $x > u - \in$.

(b) Show that if $\{x_n\}$ is a convergent sequence and $c \in \mathbb{R}$, then $\lim_{n\to\infty} cx_n = c \lim_{n\to\infty} x_n$.

(c) Prove that if $|a| < 1$, then $\lim_{n\to\infty} a^n = 0$.

3. (a) Prove that a sequence cannot converge to more than one limit.

(b) Show that $\lim_{n\to\infty} n^{\frac{1}{n}} = 1$.

(c) Show that the sequence $\langle f_n \rangle$ where

$$f_n = 1 + \frac{1}{3} + \frac{1}{3^2} + \cdots \cdots \frac{1}{3^{n-1}},$$ converges. Find $\lim_{n\to\infty} f_n$.

4. (a) Prove that a monotone sequence converges if and only if it is bounded.

(b) Show that the sequence $\langle s_n \rangle$, where $s_n = \frac{(-1)^{n-1}}{n}$ converges to zero.

(c) Prove that $\lim_{n\to\infty}\left(1 + \frac{1}{n}\right)^n$ exists and lie between 2 and 3.

5. (a) State Cauchy's n^{th} Root test for the convergence of a positive term series. Apply it to test for convergence the series

$$\left(\frac{2^2}{1^2} - \frac{2}{1}\right)^{-1} + \left(\frac{3^3}{2^3} - \frac{3}{2}\right)^{-2} + \left(\frac{4^4}{3^4} - \frac{4}{3}\right)^{-1} + \cdots.$$

(b) Test for convergence the series

(i) $\sum \sin \frac{1}{n^2}$

(ii) $\sum \frac{1}{\sqrt{n}} \tan \frac{1}{n}$